Biscovey Nursery and Infants' Teaching for Mastery Calculation Policy

The National Curriculum 2014

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics and a sense of enjoyment and curiosity about the subject.

The aims of this policy

Mastery in mathematics is for all, and the aim of this policy is to ensure all children leave our school with a secure understanding of the four operations and can confidently use and apply both written and mental calculation strategies in a range of contexts. It aims to ensure consistent strategies, models and images are used across the school to embed and deepen children's learning and understanding of mathematical concepts.

How should this policy be used?

This policy has been designed to support the teaching and planning of mathematics in our school. The policy only details the strategies, and teachers must plan opportunities for pupils to apply these; for example, when solving problems, or where opportunities emerge elsewhere in the curriculum, e.g. during outdoor learning or in the teaching kitchen. The examples and illustrations are not exhaustive but provide and overall picture of what the mathematics in our school should look like. This is not a scheme of work and must be used in conjunction with our school maths policy and national curriculum documents.

This policy sets out the progression of strategies and written methods which children will be taught as they develop in their understanding of the four operations. Strategies are set out in a Concrete, Pictorial, Abstract (CPA) approach to develop children's deep understanding and mastery of mathematical concepts. Children use concrete objects to help them make sense of the concept or problem; this could be anything from real or plastic fruit, to straws, counters or cubes. This is then developed through the use of images, models and children's own pictorial representations before moving on to the abstract mathematics. Children will travel along this continuum again and again, often revisiting previous stages when a concept is extended. It is also worth noting that if a child has moved on from the concrete to the pictorial, it does not mean that the concrete cannot be used alongside the pictorial. Or if a child is working in the abstract, 'proving' something or 'working out' could involve use of the concrete or pictorial.

Although the strategies are taught in a progressive sequence, they are designed to equip children with a 'tool box' of skills and strategies that they can apply to solve problems in a range of contexts. So as a new strategy is taught it does not necessarily supersede the previous, but builds on prior learning to enable children to have a variety of tools to select from. As children become increasingly independent, they will be able to and must be encouraged to select those strategies which are most efficient for the task.

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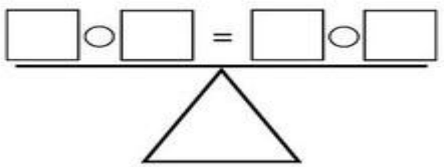
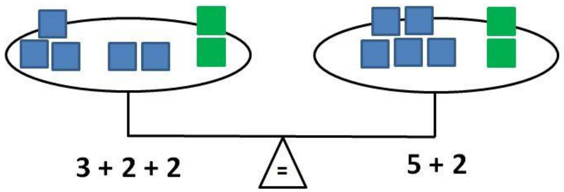
The strategies are separated into the 4 operations for ease of reference. However, it is intended that addition and subtraction, and multiplication and division will be taught together to ensure that children are making connections and seeing relationships in their mathematics. Therefore, some strategies will be taught simultaneously, for example, counting on (addition) and counting back (subtraction).

Children should be moved through the strategies at a pace appropriate to their age related expectations as defined in the EYFS Development Matters and 2014 National Curriculum for Key Stage 1. Effective teaching of the strategies relies on increasing levels of number sense, fluency and ability to reason mathematically. Children must be supported to gain depth of understanding within the strategy through the CPA approach and not learn strategies as a procedure without conceptual understanding.

Teaching equality

It is important that when teaching the 4 operations that equality (=) is also taught appropriately. Misconceptions that = means that children must 'do something' and that it indicates that an answer is required are common and must be addressed early on. Teachers should present children with number sentences and problems which place the = sign in different positions, different context and include missing box problems. For example, ?+4=7; 7=3+?; <, >, or = 5+6__ 7+4. In the concrete phase scales and Numicon provide a useful resource to demonstrate equality. Pictorial representations of equality can be used as shown below:





Importance of vocabulary

The 2014 National Curriculum places great emphasis on the importance of pupils using the correct mathematical language as a central part of their learning. Children will be unable to articulate their mathematical reasoning if they lack the mathematical vocabulary required to do so. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers modelling and only accepting what is correct. For example:

✓	×
ones	units
is equal to	equals
zero	oh (the letter O)
number sentences	sums

End of year expectations for calculations.

<table>
<tr><th>EYFS</th><th>Year 1</th><th>Year 2</th></tr>
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count reliably with numbers from one to 20.
place numbers in order.
say which number is one more or one less than a given number.
using quantities and objects, they add two single-digit numbers and count on to find the answer.
using quantities and objects, they subtract two single-digit numbers and count back to find the answer.
solve problems, including doubling, halving and sharing.

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read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs
represent and use number bonds and related subtraction facts within 20
add and subtract one-digit and two-digit numbers to 20, including zero
solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ? – 9.
solve one-step problems involving multiplication and division, by calculating the answer using concrete objects
solve one-step problems involving multiplication and division using pictorial representations and arrays with the support of the teacher

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<td>

solve problems with addition and subtraction:
using concrete objects and pictorial representations, including those involving numbers, quantities and measures
applying their increasing knowledge of mental and written methods
recall and use addition and subtraction facts to 20 fluently
derive and use related facts up to 100
add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

a two-digit number and ones
a two-digit number and tens
two two-digit numbers
adding three one-digit numbers

show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems
recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

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Progression in Calculations

Addition

Mathematics is embedded in the ethos of the Early Years where we seize every opportunity to develop children’s mathematical development and their use of mathematical vocabulary. Much of this is done through exposure to daily routines, well planned learning opportunities based on children’s individual needs and high quality interactions between children and adults.

Nursery

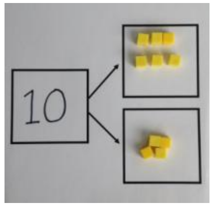
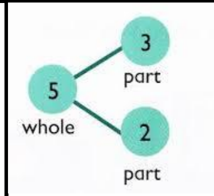
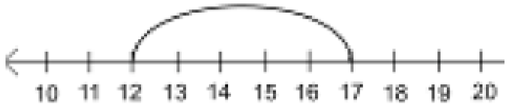
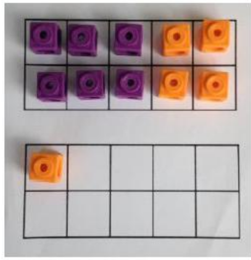
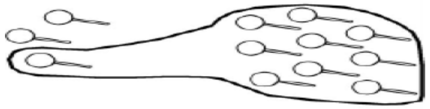
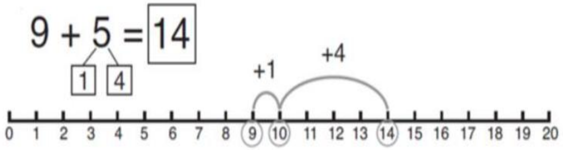
Before addition can be introduced, children need to have a secure knowledge of number. In Nursery, children are introduced to the concept of counting, number order and number recognition through practical activities, songs and games. Children also learn how to count 1-1 (pointing to each object as they count) and that anything can be counted, for example, claps, steps and jumps. The learning environment supports children’s mathematical development providing opportunities to count, solve real life problems and recognise numerals both indoors and outdoors. Our Early Years’ Practitioners are aware of the needs of individuals and support and challenge children accordingly.

Reception

On entry to Reception, children are assessed and learning opportunities are planned to progress children towards their next steps. We use the stages of development in the Development Matters document. Children need to have a secure knowledge of number and demonstrate this in their play before they begin to learn more formal addition strategies. However, our practitioners model the use of mathematical vocabulary from the very beginning. Children are introduced to the concept of addition through everyday life situations such as at the tuck table, in cooking sessions, practical games and activities. Daily routines such as making lunch choices, taking the register etc. provide rich opportunities for the beginnings of addition. All addition is done practically and where possible linked to the children’s interests and fascinations. It is important to remember that a practical addition problem requires 3 accurate counts in order to be successful; therefore the ability to count is fundamental. The learning environment, both inside and outdoors enables children to fully embed their knowledge and demonstrate this through both child-initiated and adult-led activities. The teacher will often model formal notation when singing number songs such as “Five Little Ducks, Ten in the Bed etc.” Children are encouraged to demonstrate their thought processes by making marks. These early marks form the foundations for more formal methods such as writing number sentences using standard mathematical notation (+, -, =)

Progression in Calculations

Addition

Strategies	Concrete	Pictorial	Abstract
Combining 2 groups to make a whole Counting sets of objects, combining then recounting using a 1:1 correspondence.	  	  3+2=5 	4 + 3 = 7 I have 4 apples and I pick 3 more, how many have I got altogether?
Counting on Pupils should be taught to start at the biggest number and count on, using this as an opportunity to Introduce the commutativity of addition.	 		5 + 12 = 17 Reinforce starting from the largest number. 7 + 3 = 10 Encourage recall of known number facts to develop fluency in mental calculations. .
Regrouping to make 10 To move on from the previous strategy, rather than counting on, children use their number bond knowledge and bridge to 10 e.g. if 4 + 6 = 10, so 4 + 7 must equal 11.	 6+5=11 Start with the bigger number and use the smaller number to make 10.	 3 + 9 = 	7p+4p=11p I have 7p, how much more do I need to make 10p. How much more do I add on now? If you know 10=7+3, what else do you know?

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Adding 3 single digits

Use this method as an opportunity to develop fluent recall and application of known number facts including bonds and doubles.

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Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.

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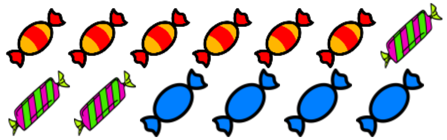

6 + 3 + 4 = 13

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$\underbrace{4 + 7 + 6}_{10} = \boxed{10} + \boxed{7}$

$= \boxed{17}$

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Partitioning to add

The emphasis for this strategy in KS1 is to develop a deep understanding of place value.

In year 2, recording addition and subtraction informally in columns supports place value and prepares for formal written methods with larger numbers later on in KS2. Ensure that when moving into any form of column the ones are calculated first.

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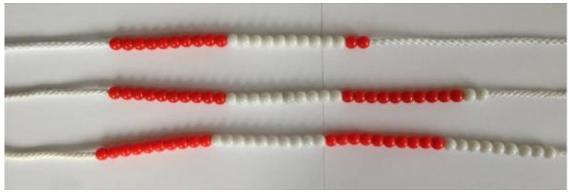

22+17=39

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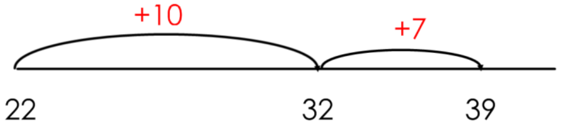
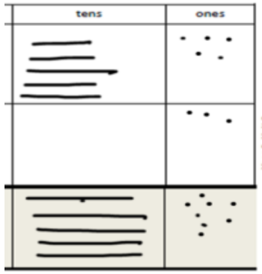
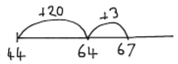


22+17=39



Begin with jumps of tens and ones then progress to jumps of all the tens and all the ones as children become more fluent.

44+23=67
20 3



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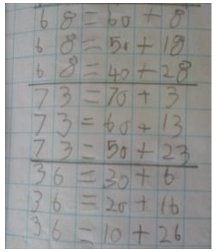
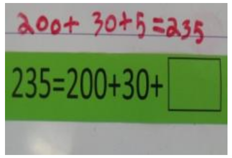
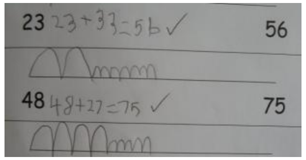




25+48=73

Informal recording in columns

20 + 5
40 + 8
60 + 13 = 73



Counting on in tens and ones to solve missing number problems

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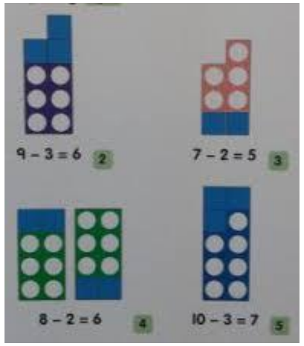
Progression in Calculations

Subtraction

Similarly to addition, subtraction is introduced explicitly when children have a sound understanding of number. However, subtraction and its associated vocabulary is introduced much earlier through experience. For example, children at the tuck table talk with an adult about the total number of raisins they have and what happens as they eat each one. These daily routines and high quality interactions between practitioner and child provide opportunities for early exploration of addition and its inverse- subtraction. When taking the register the teacher will model the writing of a subtraction sentence and use mathematical language to support children to find out how many children are present daily. Real life problem solving forms the basis of the majority of children’s mathematical development in The Early Years.

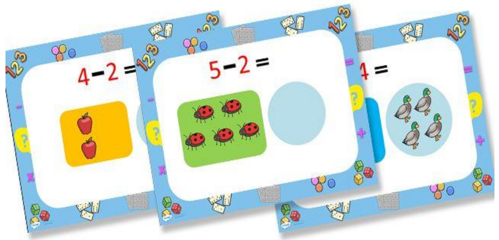


Nursery

Playful teaching and learning provides opportunities for children to be immersed in the language of mathematics. In Nursery, children begin to count forwards then backwards.This is taught through child initiated games indoors and outdoors such as acting out counting songs and running races (children shouting “5,4,3,2,1,0 - GO!”).

Reception

In its initial stages subtraction takes place in everyday play situations such as during a game of skittles or through songs such as “Ten in the Bed, Ten Green Bottles, Five Currant Buns” etc. Children use concrete objects to physically subtract a number of objects from a group. This is reinforced by opportunities provided in the outdoor area such as counting backwards to remove a specific number of fish from the water tray, singing number rhymes and subtracting or knocking down sand castles in the sand tray. Children build on their previous knowledge of ‘less/fewer’ with their understanding that subtracting means taking away a certain number of objects from a group (leaving them with less objects). Adults model subtraction vocabulary using terms suggested in Development Matters. When appropriate, adults support children in recording their subtractions in the written form in their books and on whiteboards.



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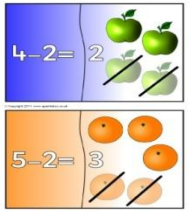
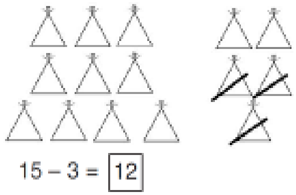
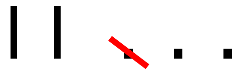
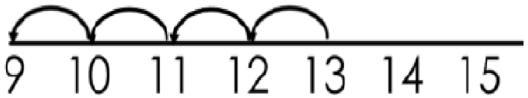
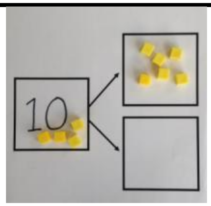
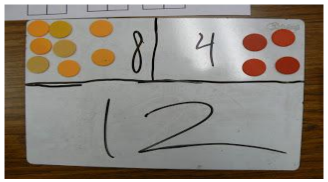
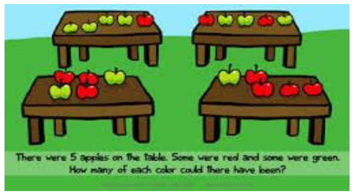
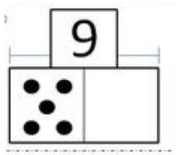
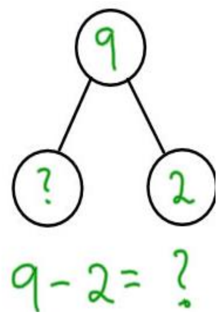
Progression in Calculations

Subtraction

Strategies	Concrete	Pictorial	Abstract
Taking away ones Use physical objects to demonstrate how something can be taken away. Move on to crossing out drawn representations. This can be developed by representing a group of ten with a line and ones with dots.	  	   23 - 1 = 12	18 -3= 15 8 – 2 = 6 There are 15 cakes in the shop. One cake is eaten, how many are left?
Counting back As with the previous, this strategy is used for subtracting small numbers from larger numbers and provides a good foundation for the concept of subtraction	 Use counters or objects and move away from the group as they are counted.	 13-4=9	Put 17 in your head, count back 5. What number are you at? Use your fingers to help.
Part, part, whole model This model develops knowledge of the inverse relationship between addition and subtraction and is used to find the answer to missing number problems.	  If 10 is the whole and 6 is one of the parts. What is the other part?	  Children should be taught the skills to approach problems in a systematic way.	 I made 9 buns for the cake sale and I only had 2 left at the end. How many did I sell? 

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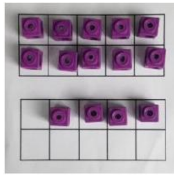
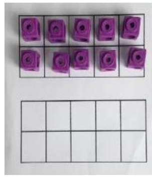
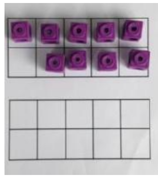
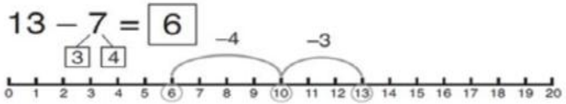
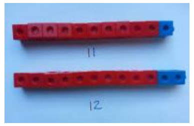
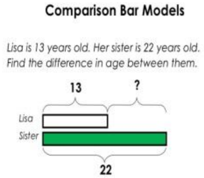
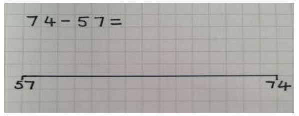
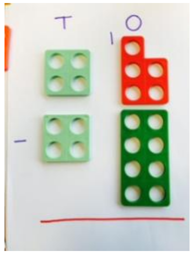
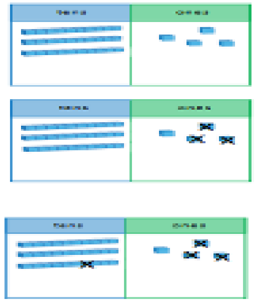
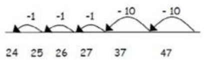
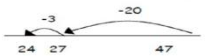
Make 10 Use this strategy to subtract a single digit number from a 2-digit number. Pupils identify how many need to be taken away to make ten first. Then they take away the rest to reach the answer.	   14-5=9 Make 14 on the ten frame or with different coloured cubes to represent the ten and the ones. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.		15 – 7= How many do we subtract to reach the next 10? How many do we have left to subtract?
Find the difference Pupils should develop a good understanding of the meaning of 'difference', exploring the inverse relationship with addition by counting back and counting up.	 Practical resources to visualise 'difference' and recognise inverse relationships e.g. 12-1=11 and 11+1=12 	  Use a blank number line to count back and count up between 2 numbers.	Lexie has 5 more strawberries than Jake. Jake has 11 cherries. How many does Lexie have? Look at the graph. Fewer children have green eyes than blue. What is the difference?
Partitioning to subtract The emphasis for this strategy in KS1 is to develop a deep understanding of place value. When not regrouping, partitioning should be developed as a mental strategy rather than formal recording in columns.	  	34-13=21 Subtract the ones first and then the tens to prepare for formal methods later  47 - 23 = 24 Partition the second number and subtract it in tens and units, as below:  Move towards more efficient jumps block, as below: 	There are 35 children in the class and 12 are boys. How many are girls? 35-12=

Progression in Calculations

Multiplication and Division

In The Early Years we teach the concepts of multiplication and division in unison. The associated vocabulary forms part of our daily conversations with children and these discussions enable us to assess children's understanding and very quickly support them with moving them towards their next steps. An enabling environment provides opportunities for children to notice patterns and pars of objects, the wellies outside, the way the cups are stacked, blocks arranged etc. all form the foundations of early multiplication and division.

Nursery and Reception

By the end of Reception, children are expected to understand the concept of doubling, halving and sharing fairly. Very young children can easily understand the concept of sharing fairly using playdough and other toys. They will very quickly be able to divide objects between to people so they have equal amounts by using the practical method of "one for you, one for me…" It is in these daily routines and real life situations that we begin to explore the vocabulary of fractions.

Later, we begin to sing songs such as the Doubles Song and start counting in twos along the number line. Children are then introduced to the concept of doubling through practical activities using socks, gloves or other paired items. For example, "I have two pairs of socks which makes four socks altogether." Dominoes are also used where children find those with pairs on them, then add both sides to find the double's total.

The teaching kitchen provides rich opportunities for children to double, half and share equally and later in the lesson when we divide our food into halves or quarters to share it fairly with our friends. Towards the end of Reception or when children are developmentally ready we begin making collections of larger numbers of objects and dividing these into groups of ten to count more easily in tens. Constant reinforcement and opportunities to embed this understanding through ***real and relevant activities, both adult and child initiated*** form the foundations for children to build on as they progress to Key Stage 1.

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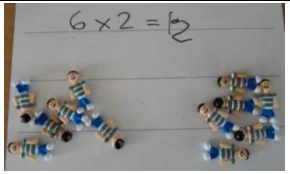
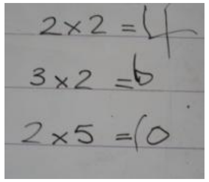
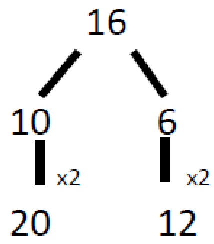
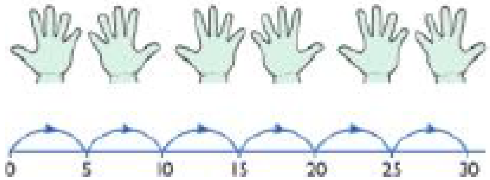
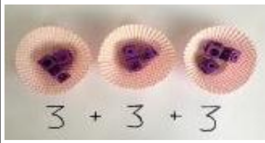
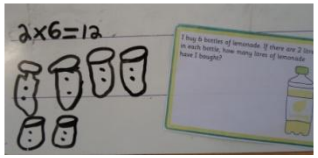
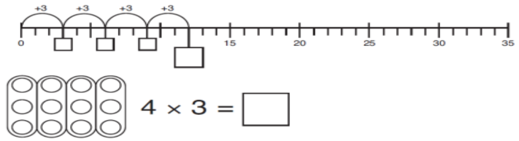
Progression in Calculations

Multiplication

Strategies	Concrete	Pictorial	Abstract
Doubling Pupils should be encouraged to develop fluent mental recall of doubles and relate to the 2 x table.		Double 4 is 8 	 If I can see 10 wheels, how many bikes are there?
Counting in multiples Pupils can use their fingers as they are skip counting, to develop an understanding of 'groups of'. Children sghould become increasingly fluent as they practise.		Use a number line or pictures to continue support in counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers and work out missing numbers in sequences both forward and backward. If I count in 2's will I get to the number 58?
Repeated addition Pupils should apply skip counting to help find the totals of repeated additions.	 5+5+5=15  3+3+3=9	 Pupils begon to recognise the relationship between repeated addition and multiplication. 	Write addition or multiplication sentences to describe objects and pictures.  2+2+2+2+2=10 2x5=10

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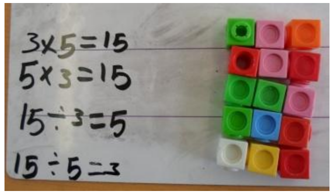
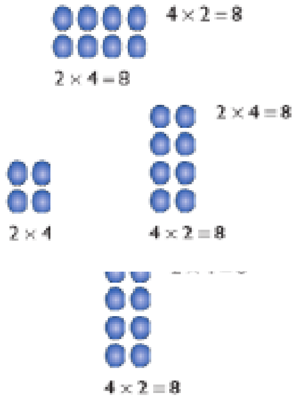
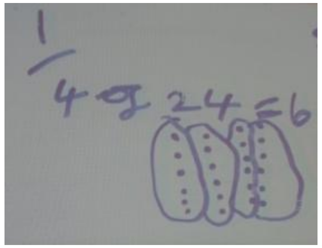
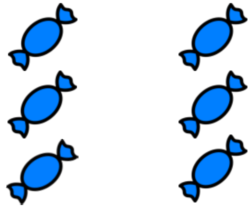
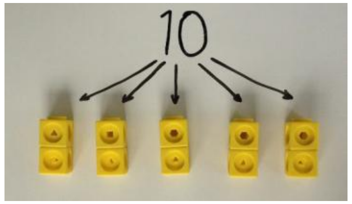
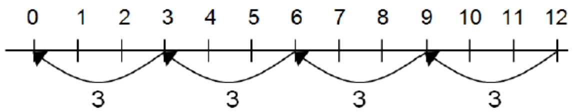
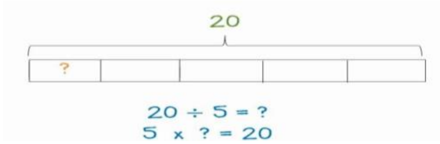
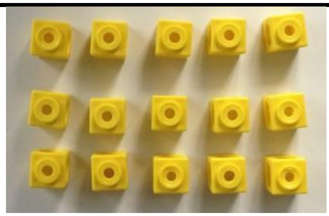
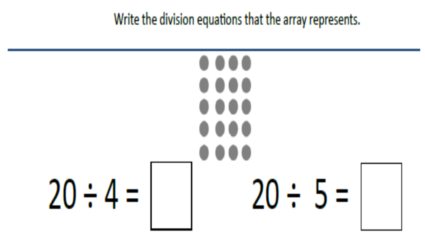
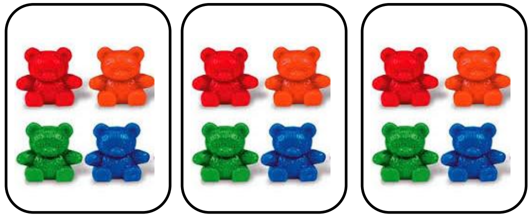
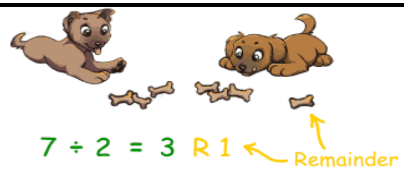
Arrays showing commutative multiplication Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	 	Draw arrays in different rotations to find **commutative** multiplication sentences.  	3 children go to the park to hunt for plne cones. They find 5 each, how many do they find altogether? 5 children eat the same number of cakes at a party. 15 cakes are eaten in total, how many did they each eat? 5+5+5=15 3x5=15 3+3+3+3+3=15 5x3=15

Progression in Calculations

Division

Strategies	Concrete	Pictorial	Abstract
Sharing Here, division is shown as sharing. E.g. If we have 24 squares of chocolate and we share them between 3 people, each person will have 8 squares each.	 	  6÷2=3	Share 9 buns between three people. 9 ÷ 3 = 3 Can you make up your own 'sharing' story and record a matching equation?
Division as grouping Here, division is shown as grouping. If we have ten cubes and put them into groups of two, there are 5 groups. This is a good opportunity to demonstrate and reinforce the inverse relationship with multiplication.	 Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.	Show jumps in groups. The number of jumps equals the number of groups.   Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group? Max is filling party bags with sweets. He has 20 sweets altogether and decides to put 5 in every bag. How many bags can he fill?
Division within arrays Use arrays of concrete manipulatives and images to find division equations. Begin to use dot arrays to develop a more abstract concept of division.		 Children can draw lines to divide their array	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

Division with a remainder This strategy provides an opportunity to reinforce prior learning of odd and even and 'multiples' when exploring how numbers can and cannot be divided into different whole numbers.	  14 ÷ 3 = Divide objects between groups and see how many are left over.	  	Complete written divisions and show the remainder using r.